IE 7039

Homework 2: Linear Programming

Due time: December 4, 2014

Total points: 100

1. Show by constructing a numerical example that the optimal solution of a linear program can be found at the vertices of the polyhedron. Your construction of the example should make use of the theorems of linear programs and the following objective function and constraints. (30 points)





Specifically, you must

1. show how you find an *arbitrary* point on the intersection of two hyperplanes of the polyhedron. (Use the first two constraints in this problem) (10 points)
2. show that the arbitrary point is dominated in the objective function value by some other points (e.g., as defined by three binding constraints). Show the systematic procedure that you use to find those points, by specifying the directional vector and computing the intersecting points. (15 points)
3. find an interior point in the feasible region. (An interior point is a point that is not bounded by any constraint.) (5 points)
4. Prove that the set of feasible solutions to the following linear program forms a convex set. (10 points)

Minimize **cTx**

Subject to **Ax=b**



1. Does the following polyhedron contain a line? If it does, give two such lines. (10 points)



1. Find a “good” separating hyperplane which can be used to classify the two datasets into two categories by using a linear program. (30 points)

These data have two real-valued attributes: x and y.

Set 1: (1, 3), (5, 7), (3, 6), (4, 5)

Set 2: (5, 2), (7, 3), (9, 5)

The objective is to maximize the minimum distance from each set of data points to the separating line.

Max Z = min of (minimum distance for set 1, minimum distance for set 2)

Note: Plot the data on 2-D space as your first step of problem analysis. You will need to use the formula for point-line distance in 2-D. (20 points)

1. A riding-mower manufacturer would like to find a way of classifying families in a city into those who are likely to purchase a riding mower（割草機）and those who are not likely to buy one. A pilot random sample of 12 owners and 11 non-owners in the city is undertaken. Find a suitable linear discrimination function. Plot the data as the first step. [Owners=1, Non-owners=2] (20 points)

|  |  |  |  |
| --- | --- | --- | --- |
| Observation | Income | Lot size 庭院 | Owner or Not |
| 1 | 60 | 18.4 | 1 |
| 2 | 85.5 | 16.8 | 1 |
| 3 | 64.8 | 21.6 | 1 |
| 4 | 61.5 | 20.8 | 1 |
| 5 | 87 | 23.6 | 1 |
| 6 | 110.1 | 19.2 | 1 |
| 7 | 108 | 17.6 | 1 |
| 8 | 82.8 | 22.4 | 1 |
| 9 | 69 | 20 | 1 |
| 10 | 93 | 20.8 | 1 |
| 11 | 51 | 22 | 1 |
| 12 | 81 | 20 | 1 |
| 13 | 52.8 | 20.8 | 2 |
| 14 | 64.8 | 17.2 | 2 |
| 15 | 43.2 | 20.4 | 2 |
| 16 | 84 | 17.6 | 2 |
| 17 | 49.2 | 17.6 | 2 |
| 18 | 59.4 | 16 | 2 |
| 19 | 66 | 18.4 | 2 |
| 20 | 47.4 | 16.4 | 2 |
| 21 | 33 | 18.8 | 2 |
| 22 | 51 | 14 | 2 |
| 23 | 63 | 14.8 | 2 |